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An Attitude Determination Algorithm for a Spacecraft Using Nonlinear Filter

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In this paper, the algorithm for a real time attitude estimation of a spacecraft motion is investigated. The proposed algorithm for attitude estimation is the second order nonlinear filter form not containing truncation error in estimation values. The proposed second order nonlinear filter has improved performance compared with the EKF (extended Kalman filter), because the algorithm does not contain any truncation bias and covariance of the estimator is compensated by the nonlinear terms of the system. Therefore, the proposed second order nonlinear filter is a suboptimal estimator. However, the proposed estimator requires a lot of computation because of an inherent nonlinearity and complexity of the system model. For more efficient computation, this paper introduces a new attitude estimation algorithm using the state divided technique for a real time processing which is developed to provide an accurate attitude determination capability under a highly maneuverable dynamic environment.

To compare the performance of the proposed algorithm with the EKF, simulations have been performed with various initial values and measurement covariances. Simulation results show that the proposed second order nonlinear algorithm outperforms the EKF. The proposed algorithm is useful for a real time attitude estimation since it has better accuracy compared with the EKF and requires less computing time compared with any existing nonlinear filters.

Key Words : Kalman Filter, Nonlinear Filter, Attitude Estimation, Spacecraft

1. Introduction

The high maneuverability requirements of a number of future three-axis slewing spacecraft, when coupled with stringent attitude and pointing accuracy requirements, demand new nonlinear filters for determining spacecraft attitude function. Moreover, the attitude determination sensors, when operating under high and continuous slew rates, acceleration, and jerk motions among possibly all three axes of the spacecraft, may introduce significant cross axis errors not otherwise encountered (Yong and Headly, 1978 ; Zwartbol, Van Den Dam, Terpstra, and Van Woerkkom, 1985; Vathsal, 1986, 1987). Another aspect of consideration is that it should operate in a real time on-board environment with only minimum ground interface in the nominal operation mode (Yong and Headly, 1978 ; Bar-Itzhack and Medan, 1983 ; Medan and Bar-Itzhack, 1985). This requirement imposes restrictions on the computational procedure of real time data reduction and processing. Based upon the above requirements, we will introduce a real time on-board precision attitude estimation to provide accurate attitude determination capability under the highly maneuverable dynamic environment.

The problem of the minimum variance estima-

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tion of the nonlinear system subjected to the stochastic noise has been applied in a wide variety of engineering problems including spacecraft attitude determination, orbit determination, space vehicle navigation, and state estimation for state vector control (Kau, Kumar and Granley, 1969 ; Jazwinski, 1970 ; Sage and Melsa, 1971). The estimation of a nonlinear system is governed by the Fokker-Planck partial differential equation, which can be derived either from the Chapman-Kolmogorov equation or the Itô differential rule (Jazwinski, 1970 ; Sage and Melsa, 1971). But, because of the infinite dimensional nature of the partial differential equation, the Fokker-Planck equation is not directly solved in any practical application. The solution of this problem necessitates seeking a finite dimensional approximation using Taylor series and Gaussian etc. (Jazwinski, 1970 ; Sage and Melsa, 1971).

Attitude information may use any sensor for which the measured quantity depends solely on the direction of some object in the sensor coordinate system (Britting, 1971 ; Lefferts, Markley and Shuster, 1982 ; LO, 1986). However, we assume that two precision star trackers exist on the spacecraft in inertial space since it is commonly used in most practical applications in attitude estimation of a spacecraft. The system concept for attitude estimation is illustrated in the block diagram shown in Fig. 1.

The spacecraft attitude model is represented by a quaternion which is a second order nonlinear system (Vathsal, 1986, 1987 ; LO, 1986). The Fokker-Planck equation corresponding to its model can be represented exactly. This paper derives the attitude estimator from the Fokker-Planck equation and the measurement model where the nonlinear terms of measurement represent the measurement noise. We will show that the proposed estimator for the spacecraft attitude does not produce any truncation bias errors, and the covariance of the estimator is compensated by the nonlinear terms of the system. The proposed estimator however requires a lot of computation because of the inherent nonlinearity and complexity of the system model for attitude. To reduce computation, this paper introduces a new attitude estimation algorithm using the state divided technique.

In the next section, a nonlinear measurement model for a star sensor will be derived. In section 3, we will discuss the special features of the propagation equation for a nonlinear attitude filter. The measurement update equation will be derived in section 4. In section 5, a new algorithm for the spacecraft attitude estimation will be developed. In section 6, we will discuss the simulation results for the proposed algorithm. Finally, in section 7, the main topics of the work will be summarized.

2. Model of Measurement Sensors

The attitude sensors considered here are star trackers, which are the most accurate in the practical applications of filters for attitude estima-

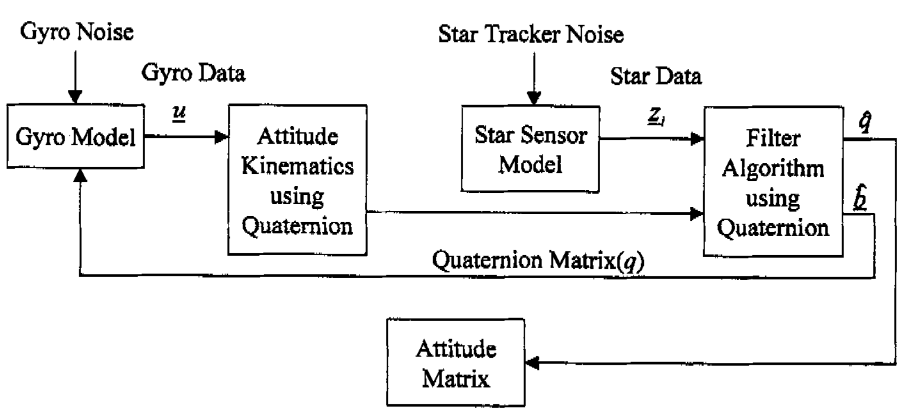

Fig. 1 Block diagram of the attitude estimation using the quaternion.

tion (Heller, 1975). Therefore, it is assumed that two precise star trackers exist on the spacecraft to obtain accurate attitude measurements of the spacecraft with respect to inertial space. The star trackers provide attitude information, and uniquely determine the sensed star unit vector in the star local frame with respect to inertial space. Between star sensor update intervals, an estimation of the spacecraft attitude is maintained by use of three rate integrating gyros. The errors in gyro output are bounded when the filter for attitude estimation is innovated by the star sensor. The measured value of a star sensor in body frame can be written as *Eq*. (1) (Heller, 1975 ; Yong and Headly, 1978).

$$\underline{z}=T\ A_{BS}(q_c)A_{SI}(q)\underline{\rho}+\underline{v}' \tag{1}$$

where, $A_{SI}(q)$ is the spacecraft attitude matrix, $A_{BS}(q_c)$ is the alignment matrix of the star tracker, and ρ is star position vector in the reference coordinate system, and $\underline{v}'$ is a random noise vector due to the star catalog position errors and star tracker output noise. The attitude matrix with quaternion in *Eq*. (1) is a second order nonlinear function. We will attempt measurement reformulation which measurement noise implicates nonlinearity of measurement. Thus, if we replace the states $\underline{x}$ in the *Eq*. (1) with $\hat{\underline{x}}+\Delta\underline{x}$, then the measurement equation is rewritten as follows:

$$\begin{aligned}\underline{z}_i=&TA_{BS}(q_c)\{A_{SI}(\hat{\underline{x}}_1)_i\underline{\rho}_i+H_i(\hat{\underline{x}}_1,\underline{\rho}_i)\Delta\underline{x}_1\\&+A_{SI}(\Delta\underline{x}_1)_i\underline{\rho}_i\}+\underline{v}_i'=TA_{BS}(q_c)\{A_{SI}(\hat{\underline{x}}_1)_i\underline{\rho}_i\\&+H_i(\hat{\underline{x}}_1,\underline{\rho}_i)\Delta\underline{x}_1\}+\underline{v}_i\end{aligned} \tag{2}$$

where, $\hat{\underline{x}}_1$ is the first term of the estimated value for $\underline{x}_1$ by Taylor series expansion, and

$$T=\begin{bmatrix}1&0&0\\0&1&0\end{bmatrix} \tag{2a}$$

$$H_i(\hat{\underline{x}}_1,\underline{\rho}_i)=\left.\frac{\partial A_{SI}(\underline{x}_1)\underline{\rho}_i}{\partial\underline{x}_1}\right|_{\underline{x}_1=\hat{\underline{x}}_1} \tag{2b}$$

$$\underline{v}=TA_{BS}(q_c)A_{SI}(\Delta\underline{x}_1)\underline{\rho}_i+\underline{v}' \tag{2c}$$

The redefined measurement noise in *Eq*. (2c) is not white Gaussian noise, but colored noise. In *Eq*. (2c), we let the elements of the star position vector be ρ_x, ρ_y, and ρ_z, respectively, then we can obtain the stochastic mean value of the colored measurement. That is,

$$\begin{aligned}E[\underline{v}]&=TA_{BS}(q_c)E[A_{SI}(\Delta\underline{x}_1)\underline{\rho}_i]\\&=TA_{BS}(q_c)\\&\begin{bmatrix}(p_{11}-p_{22}-p_{33}+p_{44})\rho_x+2(p_{12}+p_{34})\rho_y+2(p_{13}-p_{24})\rho_z\\2(p_{12}-p_{34})\rho_x+(-p_{11}+p_{22}-p_{33}+p_{44})\rho_y+2(p_{14}+p_{23})\rho_y\\2(p_{13}+p_{24})\rho_x+2(p_{23}-p_{14})\rho_y+(-p_{11}-p_{22}+p_{33}+p_{44})\rho_z\end{bmatrix}\end{aligned} \tag{3}$$

where, p_{ij} are the covariance elements corresponding to state i and j. Also, the covariance R of colored measurement noise can be calculated from *Eq*. (2c) and *Eq*. (3).

$$\begin{aligned}R=&E[\underline{v}\underline{v}^T]-E[\underline{v}]E[\underline{v}]^T\\&=TA_{BS}(q_c)\{E[A_{SI}(\Delta\underline{x}_1)\underline{\rho}\underline{\rho}^TA_{SI}(\Delta\underline{x}_1)^T]\\&\quad-A(P)\underline{\rho}\underline{\rho}^TA(P)^T\}A_{BS}(q_c)^TT^T+R'\end{aligned} \tag{4}$$

The first term in *Eq*. (4) has only fourth-order moment of state errors $\Delta\underline{x}$. If we assume that state errors $\Delta\underline{x}$ are Gaussian, then fourth order moment generally has the relation among state as *Eq*. (5) (Jazwinski, 1970).

$$E[\Delta x_i\Delta x_j\Delta x_k\Delta x_l]=p_{jk}p_{il}+p_{jl}p_{ik}+p_{kl}p_{ij} \tag{5}$$

Therefore, applying *Eq*. (3) and *Eq*. (5) into *Eq*. (4) brings *Eq*. (6) as follows:

$$R=TA_{BS}(q_c)\{L(P,\underline{\rho})\}A_{BS}(q_c)^TT^T+R' \tag{6}$$

where,

$$\begin{aligned}L(P,\underline{\rho})=&E[A_{SI}(\Delta\underline{x}_1)\underline{\rho}\underline{\rho}^TA_{SI}(\Delta\underline{x}_1)^T]\\&-A(P)\underline{\rho}\underline{\rho}^TA(P)^T\end{aligned} \tag{7a}$$

$$A_{SI}(\Delta\underline{x}_1)=\begin{bmatrix}\Delta x_1^2-\Delta x_2^2-\Delta x_3^2+\Delta x_4^2 & 2(\Delta x_1x_2+\Delta x_3\Delta x_4) & 2(\Delta x_1\Delta x_3-\Delta x_2\Delta x_4)\\2(\Delta x_1\Delta x_2-\Delta x_3\Delta x_4) & -\Delta x_1^2+\Delta x_2^2-\Delta x_3^2+\Delta x_4^2 & 2(\Delta x_1\Delta x_4+\Delta x_2\Delta x_4)\\2(\Delta x_1\Delta x_3+\Delta x_2\Delta x_4) & 2(\Delta x_2\Delta x_3-\Delta x_1\Delta x_4) & -\Delta x_1^2-\Delta x_2^2+\Delta x_3^2+\Delta x_4^2\end{bmatrix} \tag{7b}$$

$$A(P)=\begin{bmatrix}p_{11}-p_{22}-p_{33}+p_{44} & 2(p_{12}+p_{34}) & 2(p_{13}-p_{24})\\2(p_{12}-p_{34}) & -p_{11}+p_{22}-p_{33}+p_{44} & 2(p_{14}+p_{23})\\2(p_{13}+p_{24}) & 2(p_{23}-p_{14}) & -p_{11}-p_{22}+p_{33}+p_{44}\end{bmatrix} \tag{7c}$$

Substituting *Eq*. (7b) and *Eq*. (7c) in *Eq*. (7a) gives

$$L(P,\rho)=\begin{bmatrix}l_{11}&l_{12}&l_{13}\\l_{21}&l_{22}&l_{23}\\l_{31}&l_{32}&l_{33}\end{bmatrix} \tag{8a}$$

$l_{11}=\{2(p_{11}^2+p_{22}^2+p_{33}^2+p_{44}^2)+4(-p_{12}^2-p_{13}^2+p_{14}^2$

$$
\begin{aligned}
&+p_{23}^2-p_{24}^2-p_{34}^2)\}\rho_x^2+4(p_{11}p_{22}+p_{33}p_{44}+p_{12}^2\\
&+p_{34}^2+2p_{13}p_{24}+2p_{14}p_{23})\rho_y^2+4(p_{11}p_{33}+p_{22}p_{44}\\
&+p_{13}^2+p_{24}^2-2p_{12}p_{34}-2p_{14}p_{23})\rho_z^2+8(p_{11}p_{12}\\
&-p_{22}p_{12}-p_{33}p_{34}+p_{44}p_{34}-p_{31}p_{32}+p_{41}p_{42}\\
&+p_{13}p_{14}-p_{23}p_{24})\rho_x\rho_y+8(p_{11}p_{13}-p_{33}p_{13}\\
&+p_{22}p_{24}-p_{44}p_{42}-p_{21}p_{23}+p_{41}p_{43}-p_{12}p_{14}\\
&+p_{32}p_{34})\rho_x\rho_z+8(p_{11}p_{23}-p_{22}p_{14}+p_{33}p_{14}\\
&-p_{44}p_{23}+p_{12}p_{13}-p_{21}p_{24}+p_{13}p_{34}-p_{42}p_{43})\rho_y\rho_z \qquad \text{(8b)}
\end{aligned}
$$

$$
\begin{aligned}
l_{12}=&4(p_{11}p_{12}-p_{22}p_{12}+p_{33}p_{34}-p_{44}p_{34}+p_{14}p_{24}\\
&-p_{31}p_{32}-p_{13}p_{14}+p_{23}p_{24})\rho_x^2+4(-p_{11}p_{12}\\
&+p_{22}p_{12}-p_{33}p_{34}+p_{44}p_{34}+p_{14}p_{24}-p_{31}p_{32}\\
&-p_{13}p_{14}+p_{23}p_{24})\rho_y^2+4(p_{11}p_{34}+p_{33}p_{12}\\
&-p_{44}p_{12}-p_{22}p_{34}+p_{13}p_{14}+p_{31}p_{32}-p_{14}p_{24}\\
&-p_{23}p_{24})\rho_z^2+\{2(-p_{11}^2-p_{22}^2+p_{33}^2+p_{44}^2)\\
&+4(p_{11}p_{22}-p_{33}p_{44})+8(p_{12}^2-p_{34}^2)\}\rho_x\rho_y\\
&+\{4(p_{11}p_{14}+p_{44}p_{14}-p_{22}p_{23}-p_{33}p_{23}+p_{11}p_{23}\\
&-p_{22}p_{14}-p_{33}p_{14}+p_{44}p_{23})+8(p_{12}p_{13}\\
&-p_{21}p_{24}-p_{31}p_{34}+p_{42}p_{43})\}\rho_x\rho_z+\{4(-p_{11}p_{13}\\
&-p_{33}p_{13}-p_{22}p_{24}-p_{44}p_{24}+p_{11}p_{24}+p_{22}p_{13}\\
&+p_{44}p_{13}+p_{33}p_{24})+8(p_{12}p_{14}+p_{12}p_{23}+p_{14}p_{43}\\
&+p_{32}p_{34})\}\rho_y\rho_z \qquad \text{(8c)}
\end{aligned}
$$

$$
\begin{aligned}
l_{13}=&4(p_{11}p_{13}-p_{33}p_{13}-p_{22}p_{24}+p_{44}p_{24}-p_{21}p_{23}\\
&+p_{41}p_{43}+p_{14}p_{12}-p_{32}p_{34})\rho_x^2+4(p_{22}p_{13}-p_{11}p_{24}\\
&+p_{33}p_{24}-p_{44}p_{13}+p_{21}p_{23}-p_{14}p_{12}+p_{32}p_{34}\\
&-p_{43}p_{41})\rho_y^2+4(-p_{11}p_{13}+p_{33}p_{13}+p_{22}p_{24}\\
&-p_{44}p_{24}-p_{21}p_{23}+p_{14}p_{34}+p_{12}p_{14}-p_{32}p_{34})\rho_z^2\\
&+\{4(-p_{22}p_{23}-p_{33}p_{23}-p_{11}p_{14}-p_{44}p_{14}+p_{11}p_{23}\\
&+p_{44}p_{23}+p_{22}p_{14}+p_{33}p_{14})+8(p_{12}p_{13}+p_{42}p_{43}\\
&+p_{21}p_{24}+p_{31}p_{34})\}\rho_x\rho_y+\{2(-p_{11}^2+p_{22}^2-p_{33}^2\\
&+p_{44}^2)+4(p_{11}p_{23}-p_{22}p_{44})+8(p_{13}^2-p_{24}^2)\}\rho_x\rho_z\\
&+\{4(-p_{11}p_{12}-p_{22}p_{12}+p_{33}p_{34}+p_{44}p_{34}+p_{33}p_{12}\\
&+p_{44}p_{12}-p_{11}p_{34}-p_{22}p_{34})+8(p_{31}p_{32}+p_{41}p_{42}\\
&-p_{41}p_{43}-p_{24}p_{23})\}\rho_y\rho_z \qquad \text{(8d)}
\end{aligned}
$$

$$l_{21}=l_{12} \qquad \text{(8e)}$$

$$
\begin{aligned}
l_{22}=&\{4(p_{11}p_{22}+p_{33}p_{44}+p_{12}^2+p_{34}^2)+8(-p_{13}p_{24}\\
&-p_{14}p_{23})\}\rho_x^2+\{2(p_{11}^2+p_{22}^2+p_{33}^2+p_{44}^2)+4(-p_{12}^2\\
&+p_{13}^2-p_{14}^2+p_{24}^2-p_{23}^2-p_{34}^2)\}\rho_y^2+\{4(p_{11}p_{44}\\
&+p_{22}p_{33})+4(p_{14}^2+p_{23}^2)+8(p_{12}p_{34}+p_{13}p_{24})\}\rho_z^2\\
&+8(p_{22}p_{12}-p_{11}p_{12}+p_{33}p_{44}-p_{44}p_{34}-p_{31}p_{32}\\
&+p_{41}p_{42}+p_{13}p_{14}-p_{23}p_{24})\rho_x\rho_y+8(p_{11}p_{24}\\
&+p_{22}p_{13}-p_{44}p_{13}-p_{33}p_{24}+p_{12}p_{14}+p_{21}p_{23}\\
&-p_{41}p_{43}-p_{32}p_{34})\rho_x\rho_z+8(-p_{11}p_{14}+p_{44}p_{14}\\
&+p_{22}p_{23}-p_{33}p_{23}+p_{21}p_{24}-p_{31}p_{34}-p_{12}p_{13}\\
&+p_{42}p_{43})\rho_y\rho_z \qquad \text{(8f)}
\end{aligned}
$$

$$
\begin{aligned}
l_{23}=&4(p_{11}p_{23}+p_{22}p_{14}-p_{33}p_{14}-p_{44}p_{23}+p_{12}p_{13}\\
&+p_{21}p_{24}-p_{31}p_{34}-p_{42}p_{43})\rho_x^2+4(-p_{32}p_{33}\\
&+p_{11}p_{14}+p_{22}p_{23}-p_{44}p_{14}+p_{31}p_{34}-p_{21}p_{24}\\
&-p_{13}p_{12}+p_{42}p_{43})\rho_y^2+4(-p_{11}p_{14}+p_{44}p_{14}\\
&-p_{22}p_{23}+p_{33}p_{32}-p_{21}p_{24}+p_{31}p_{34}-p_{12}p_{13}\\
&+p_{42}p_{43})\rho_z^2+\{4(-p_{11}p_{13}-p_{33}p_{13}+p_{22}p_{24}\\
&+p_{44}p_{24}+p_{22}p_{13}-p_{11}p_{24}-p_{33}p_{24}+p_{44}p_{13})\\
&+8(p_{12}p_{23}-p_{12}p_{14}-p_{32}p_{34}+p_{41}p_{43})\}\rho_x\rho_y\\
&+\{4(-p_{11}p_{12}-p_{22}p_{12}-p_{33}p_{34}-p_{44}p_{34})\\
&+6(p_{33}p_{12}+p_{44}p_{12}+p_{11}p_{34}+p_{22}p_{34})+8(p_{13}p_{32}\\
&+p_{41}p_{42}+p_{13}p_{14}+p_{23}p_{24})\}\rho_x\rho_z+\{2(p_{11}^2-p_{22}^2\\
&-p_{33}^2+p_{44}^2)+4(p_{22}p_{33}-p_{11}p_{44})+8(p_{23}^2\\
&-p_{14}^2)\}\rho_y\rho_z \qquad \text{(8g)}
\end{aligned}
$$

$$l_{31}=l_{13} \qquad \text{(8h)}$$

$$l_{32}=l_{23} \qquad \text{(8i)}$$

$$
\begin{aligned}
l_{33}=&\{4(p_{11}p_{33}+p_{22}p_{44}+p_{13}^2+p_{24}^2)+8(p_{12}p_{34}\\
&+p_{14}p_{32})\}\rho_x^2+\{4(p_{22}p_{33}+p_{11}p_{44}+p_{23}^2+p_{14}^2)\\
&+8(-p_{12}p_{34}-p_{13}p_{24})\}\rho_y^2+\{2(p_{11}^2+p_{22}^2+p_{33}^2\\
&+p_{44}^2)+4(p_{12}^2-p_{13}^2-p_{14}^2-p_{23}^2-p_{24}^2+p_{34}^2)\}\rho_z^2\\
&+8(p_{33}p_{12}-p_{11}p_{34}+p_{22}p_{34}-p_{44}p_{12}+p_{31}p_{32}\\
&-p_{13}p_{14}+p_{23}p_{24}-p_{14}p_{24})\rho_x\rho_y+8(p_{11}p_{13}\\
&+p_{33}p_{13}-p_{22}p_{24}+p_{44}p_{24}-p_{21}p_{23}+p_{41}p_{43}\\
&-p_{12}p_{14}+p_{32}p_{34})\rho_x\rho_z+8(-p_{22}p_{23}+p_{33}p_{32}\\
&+p_{11}p_{14}-p_{44}p_{14}-p_{12}p_{13}+p_{42}p_{43}+p_{21}p_{24}\\
&-p_{31}p_{34})\rho_y\rho_z \qquad \text{(8j)}
\end{aligned}
$$

Note only that the derived measurement equation (*Eq.* (2)) subject to *Eq.* (3) and *Eq.* (6) can be very easily determined from the star position vector, and recursive formulas are then readily applicable because of a unique feature quaternion expression of a measurement equation. By observing the measurement equation in *Eq.* (2), one can see that we intend to correct the model error due to the nonlinear terms of the measurement.

3. Propagation of the Filter for Attitude Estimation

The gyro errors are caused by misalignment and uncertainty, which contains bias, mass-unbalance, anisoelasticity and scale factor (Heller, 1975). We assume that gyro errors are compensated by the gyro calibration, then gyro remaining errors are only bias and white noise. Therefore, the gyro measurement equation is modeled as follows:

$$\underline{u}(t)=\underline{\omega}(t)+\underline{b}(t)+\eta_1(t) \qquad (9)$$

where, the vector $\underline{\omega}$ is the true angular velocity, $\underline{b}$ is the drift rate bias and $\underline{\eta}_1$ is the drift rate noise. The $\underline{\eta}_1$ is assumed to be a Gaussian white noise process.

$$E[\underline{\eta}_1(t)]=\underline{0} \tag{10a}$$
$$E[\underline{\eta}_1(t)\underline{\eta}_1(t')^T]=Q_1(t)\delta(t-t') \tag{10b}$$

The drift rate bias is itself not a static quantity but is driven by a second-Gaussian white noise process, and the two noise processes are assumed to be uncorrelated as follows:

$$\dot{\underline{b}}(t)=\underline{\eta}_2(t) \tag{11a}$$
$$E[\underline{\eta}_2(t)]=\underline{0} \tag{11b}$$
$$E[\underline{\eta}_2(t)\underline{\eta}_2(t')^T]=Q_2(t)\delta(t-t') \tag{11c}$$
$$E[\underline{\eta}_1(t)\underline{\eta}_2(t')^T]=0 \tag{11d}$$

Attitude determination of the spacecraft involves the estimation of the orientation of the spacecraft axes in space (Britting, 1971). This is achieved by processing data from on-board or ground station sensors. Commonly used attitude estimation methods for a spacecraft are the Euler method, the direction cosine method, and the quaternion method (Miller, 1978 ; Nurse, 1978 ; Bar-Itzhack and Oshman, 1985 ; Bar-Itzhack and Iden, 1987). Among them, the quaternion method is the most popular because of its advantages in nonsingularity, simplicity, and computation time (Miller, 1978 ; Nurse, 1978). In the system investigated, the attitude is represented by the quaternion defined as:

$$q=\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}=\begin{bmatrix} \phi_x/\phi_0\sin(\phi_0/2) \\ \phi_y/\phi_0\sin(\phi_0/2) \\ \phi_z/\phi_0\sin(\phi_0/2) \\ \cos(\phi_0/2) \end{bmatrix} \tag{12}$$

where, the vector $\underline{\phi}$ is the rotational unit vector related to the rotation axes and the angle ϕ_0 is the magnitude of the rotational vector. The quaternion possesses three degrees of freedom and satisfies the constraint (Miller, 1978).

$$q^Tq=1 \tag{13}$$

The differential equation for the quaternion is given by (Miller, 1978 ; Nurse, 1978)

$$\dot{q}=1/2\Omega(\underline{\omega})q \tag{14a}$$

where, $\Omega(\underline{\omega})$ is the skew symmetric matrix given by

$$\Omega(\underline{\omega})=\begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix} \tag{14b}$$

We assume that the system states for the spacecraft attitude are given by the attitude quaternion and the gyro drift rate bias vector, then the attitude system is of dimension seven.

$$\underline{x}(t)\equiv\begin{bmatrix} \underline{x}_1(t) \\ \underline{x}_2(t) \end{bmatrix}\equiv\begin{bmatrix} q(t) \\ \underline{b}(t) \end{bmatrix} \tag{15}$$

The quaternion and the bias vector have been shown to satisfy the coupled differential equations (Miller, 1978 ; Lefferts, 1982).

$$\dot{q}(t)=1/2\Omega(\underline{u}(t)-\underline{b}(t)-\underline{\eta}_1(t))q(t) \tag{16a}$$
$$\dot{\underline{b}}(t)=\underline{\eta}_2(t) \tag{16b}$$

Let us express the errors of the state $\underline{x}$ as follows:

$$\begin{bmatrix} \Delta\underline{x}_1(t) \\ \Delta\underline{x}_2(t) \end{bmatrix}=\begin{bmatrix} q(t)-\hat{q}(t) \\ \underline{b}(t)-\hat{\underline{b}}(t) \end{bmatrix} \tag{17}$$

which is an implied definition of $\Delta\underline{x}$. Substitution of $Eq.$ (17) into $Eq.$ (16) yields:

$$\begin{aligned}\begin{bmatrix} \dot{\underline{x}}_1(t) \\ \dot{\underline{x}}_2(t) \end{bmatrix}&=\begin{bmatrix} 1/2\Omega(\underline{u}-\hat{\underline{x}}_2(t)) & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \hat{\underline{x}}_1(t) \\ \hat{\underline{x}}_2(t) \end{bmatrix}\\&+\begin{bmatrix} 1/2\Omega(\underline{u}-\hat{\underline{x}}_2(t)) & -1/2\Gamma(\hat{\underline{x}}_1(t)) \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \Delta\underline{x}_1(t) \\ \Delta\underline{x}_2(t) \end{bmatrix}\\&+\begin{bmatrix} -1/2\Gamma(\hat{\underline{x}}_1(t)) & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} \underline{\eta}_1(t) \\ \underline{\eta}_2(t) \end{bmatrix}\\&+\begin{bmatrix} -1/2\Gamma(\Delta\underline{x}_1(t)) & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \underline{\eta}_1(t) \\ \underline{\eta}_2(t) \end{bmatrix}\\&+\begin{bmatrix} -1/2\Omega(\Delta\underline{x}_2(t))\Delta\hat{\underline{x}}_1(t) \\ 0 \end{bmatrix}\end{aligned} \tag{18}$$

When we apply $Eq.$ (18) into the propagation equation derived from the Fokker-Planck equation, the state propagation equation is exactly obtained as follows (Sage and Melsa, 1971):

$$\begin{aligned}\dot{\hat{\underline{x}}}&=E[f(\underline{x})]\\&=\begin{bmatrix} 1/2\Omega(\underline{u}-\hat{\underline{x}}_2(t)) & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \hat{\underline{x}}_1(t) \\ \hat{\underline{x}}_2(t) \end{bmatrix}\\&+E\begin{bmatrix} -1/2\Omega(\Delta\underline{x}_2(t))\Delta\hat{\underline{x}}_1(t) & 0 \\ 0 & 0 \end{bmatrix}\end{aligned} \tag{19a}$$

where, $f(\underline{x})$ is the right-half term of $Eq.$ (18), and

$$E[-1/2\Omega(\Delta \underline{x}_2(t))\Delta \underline{x}_1(t)] = \frac{1}{2}\begin{bmatrix} -p_{27}+p_{36}-p_{45} \\ p_{17}-p_{35}-p_{46} \\ -p_{16}+p_{25}-p_{47} \\ p_{15}+p_{26}+p_{37} \end{bmatrix} \quad (19b)$$

The prediction values of the state vector are obtained from *Eq.* (19) with the special propagation structure having properties that do not contain truncation errors due to the nonlinearity of the system for attitude dynamics.

Also, the differential equation of seven dimensional error covariance matrix P is exactly derived from the Fokker-Planck equation as follows:

$$\dot{P} = E[f\Delta \underline{x}^T] + E[\Delta \underline{x} f^T] + E[GQG^T] \quad (20)$$

The solution of *Eq.* (20) is employed by expanding the function $f(x, t)$ by the Taylor series, but the Taylor series method used to solve *Eq.* (20) (Jazwinski, 1970 ; Sage and Melsa, 1971) is difficult and complex. If we let the state errors be Gaussian, then the continuous propagation equation of covariance P is obtained by the substitution of *Eq.* (18) into *Eq.* (20).

$$\dot{P}(t) = P(t)F^T + FP(t) + GQG^T + M \quad (21a)$$

where,

$$F(t) = \begin{bmatrix} 1/2\Omega(\underline{u}(t) - \hat{\underline{x}}_2(t)) & -1/2\Gamma(\hat{\underline{x}}_1(t)) \\ 0 & 0 \end{bmatrix} \quad (21b)$$

$$\Gamma(\underline{x}) = \begin{bmatrix} x_4 & -x_3 & x_2 \\ x_3 & x_4 & -x_1 \\ -x_2 & x_1 & x_4 \\ -x_1 & -x_2 & -x_3 \end{bmatrix} \quad (21c)$$

$$G(t) = \begin{bmatrix} -1/2\Gamma(\hat{\underline{x}}_1(t)) & 0 \\ 0 & I \end{bmatrix} \quad (21d)$$

$$M(t) = E\begin{bmatrix} 1/4\Gamma(\Delta \underline{x}_1(t))Q_1\Gamma^T(\Delta \underline{x}_1(t)) & 0 \\ 0 & 0 \end{bmatrix} \quad (21e)$$

When we derive *Eq.* (21), we neglect the third order moment of state errors since they are assumed to be a Gaussian process. Although the state errors are not Gaussian, a third order moment generally is nearly zero because the probability density function has symmetric properties in spite of being non-Gaussian. Therefore, the filter gain obtained from the solution of *Eq.* (21) will be a suboptimal gain regardless of being non-Gaussian.

Since the system matrix F in *Eq.* (21) is a singular matrix, it is difficult to propagate the error in the seven dimensional covariance matrix P. A transformation matrix shown in Lefferts, Markley and Shuster (1982) propagates the error covariance matrix in six dimensional state space. The transformation matrix S is given by

$$S(\hat{\underline{x}}_1) = \begin{bmatrix} \Gamma(\hat{\underline{x}}_1(t)) & 0 \\ 0 & I \end{bmatrix}_{7\times 6} \quad (22)$$

The six dimensional error covariance matrix, denoted as P', is given by

$$P'(t) = S^T P(t) S \quad (23a)$$

$$P(t) = SP'(t)S^T \quad (23b)$$

The covariance differential equation of six dimensions is derived from differentiating Eq. (23) and using the properties of the S matrix into *Eq.* (21). This result, the covariance differential equation of six dimensions, is given by *Eq.* (24).

$$\dot{P}' = F'P' + P'(F')^T + G'Q(G')^T + M' \quad (24a)$$

where,

$$F'(t) = \begin{bmatrix} [\hat{\omega}(t)x] & -1/2I \\ 0 & 0 \end{bmatrix} \quad (24b)$$

$$G'(t) = \begin{bmatrix} -1/2I & 0 \\ 0 & I \end{bmatrix} \quad (24c)$$

$$[\underline{\omega}(t)x] = \begin{bmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{bmatrix} \quad (24d)$$

$$M' = \begin{bmatrix} 1/4E[\Gamma^T(\hat{\underline{x}}_1)\Gamma(\Delta \underline{x}_1)Q_1\Gamma^T(\Delta \underline{x}_1)\Gamma(\hat{\underline{x}}_1)] & 0 \\ 0 & 0 \end{bmatrix} \quad (24e)$$

where, $[\underline{\omega}(t)x]$ (Britting, 1971) is the skew symmetric matrix.

To compute M' in *Eq.* (24), we must first compute M. If matrix Q_1 is assumed to be a diagonal matrix, then it can be shown that

$$E[\Gamma(\Delta \underline{x}_1)Q_1\Gamma^T(\Delta \underline{x}_1)] = \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} \\ \beta_{31} & \beta_{32} & \beta_{33} & \beta_{34} \\ \beta_{41} & \beta_{42} & \beta_{43} & \beta_{44} \end{bmatrix}$$

$$
=\begin{bmatrix}
\alpha_1 p_{44}+\alpha_2 p_{33}+\alpha_3 p_{22} & (\alpha_1-\alpha_2)p_{34}-\alpha_3 p_{12} & (\alpha_3-\alpha_1)p_{24}-\alpha_2 p_{13} & (\alpha_2-\alpha_3)p_{23}-\alpha_1 p_{14} \\
(\alpha_1-\alpha_2)p_{34}-\alpha_3 p_{12} & \alpha_1 p_{33}+\alpha_2 p_{44}+\alpha_3 p_{11} & (\alpha_2-\alpha_3)p_{14}-\alpha_1 p_{23} & (\alpha_3-\alpha_1)p_{13}-\alpha_2 p_{24} \\
(\alpha_3-\alpha_1)p_{24}-\alpha_2 p_{13} & (\alpha_2-\alpha_3)p_{14}-\alpha_1 p_{23} & \alpha_1 p_{22}+\alpha_1 p_{11}+\alpha_3 p_{44} & (\alpha_1-\alpha_2)p_{12}-\alpha_3 p_{34} \\
(\alpha_2-\alpha_3)p_{23}-\alpha_1 p_{14} & (\alpha_3-\alpha_1)p_{13}-\alpha_2 p_{24} & (\alpha_1-\alpha_2)p_{12}-\alpha_3 p_{34} & \alpha_1 p_{11}+\alpha_2 p_{22}+\alpha_3 p_{33}
\end{bmatrix} \tag{25}
$$

where, α_1, α_2, and α_3 are the elements of the covariance matrix Q_1 for the gyro measurement noise on three axes. The six dimensional matrix M' can be obtained by substitution of $Eq.$ (25) into the first block matrix of $Eq.$ (24e).

$$
1/4M'(t)=1/4\Gamma^T(\underline{\hat{x}}_1)\begin{bmatrix}
\beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\
\beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} \\
\beta_{31} & \beta_{32} & \beta_{33} & \beta_{34} \\
\beta_{41} & \beta_{42} & \beta_{43} & \beta_{44}
\end{bmatrix}\Gamma(\underline{\hat{x}}_1) \tag{26}
$$

But $Eq.$ (26) is not in recursive form because the derived β contains the covariance elements of the seven dimensional P. To obtain the recursive form, we apply the $Eq.$ (23) into $Eq.$ (26), the first block matrix M' of $Eq.$ (24) can be obtained as follows:

$$
M'(t)=\begin{bmatrix}
\alpha_2 p'_{33}+\alpha_3 p'_{22} & -\alpha_3 p'_{12} & -\alpha_2 p'_{13} \\
-\alpha_3 p'_{12} & \alpha_1 p'_{33}+\alpha_3 p'_{11} & -\alpha_1 p'_{23} \\
-\alpha_2 p'_{13} & -\alpha_1 p'_{23} & \alpha_2 p'_{11}+\alpha_1 p'_{22}
\end{bmatrix} \tag{27}
$$

Substituting $Eq.$ (27) into $Eq.$ (24e), we can obtain the recursive form of the six dimensional covariance differential equation. Comparing it with the derived six dimensional covariance equation by Vathsal (1986, 1987), we can find that the derived six dimensional covariance equation is in recursive form for the general gyro model.

4. Discrete Update Equation

The discrete state update equation can be defined as $Eq.$ (28).

$$
\underline{\hat{x}}_k(+)=\underline{\hat{x}}_k(-)+K_k[\underline{z}_k-\underline{\hat{z}}_k] \tag{28}
$$

Substituting $Eq.$ (2) into $Eq.$ (28), the update equation is rewritten as follows:

$$
\underline{\hat{x}}_k(+)=\underline{\hat{x}}_k(-)+K_k\{\underline{z}_k-TA_{BS}(q_c)A_{SI}(\underline{\hat{x}}_1)_i\underline{\rho}_i-E[\underline{v}_i]\}=\underline{\hat{x}}_k(-)+K_k\{\underline{z}_k-h_k(\underline{\hat{x}}_1)-\pi_k\} \tag{29}
$$

where, $h_k(\underline{\hat{x}}_1)=TA_{BS}(q_c)A_{SI}(\underline{\hat{x}}_1)_i\underline{\rho}_i$ and π_k is the mean of measurement noise that includes the nonlinear terms. We already know it from $Eq.$ (19) and $Eq.$ (2), that $x_k(-)$ in $Eq.$ (29) does not have truncation error and the residual of measurement also does not have truncation error. Therefore, the derived update equation ($Eq.$ (29)) does not have truncation error due to nonlinear terms in the system and in the measurement. Let us define the update covariance P as $Eq.$ (30).

$$
P_k(+)=E[\Delta\underline{x}_k(+)\Delta\underline{x}_k^T(+)] \tag{30}
$$

where, $\Delta\underline{x}_k(+)$ is the state error vector. Following $Eq.$ (2) and $Eq.$ (19), the state error equation can be written as:

$$
\begin{aligned}
\Delta\underline{x}_k(+) &= \underline{x}_k-\underline{\hat{x}}_k(+) \\
&= \Delta\underline{x}_k(-)-K_k\{TA_{BS}(q_c)H_k(\underline{\hat{x}}_k,\underline{\rho}_k)\Delta\underline{x}_k+\underline{v}_k-\pi_k\}
\end{aligned} \tag{31}
$$

Substituting $Eq.$ (31) into $Eq.$ (30), we can obtain the update covariance equation as follows:

$$
\begin{aligned}
P_k(+)=E[&\Delta\underline{x}_k(-)\Delta\underline{x}_k^T(-)-\Delta\underline{x}_k(-)\Delta\underline{x}_k^T(-)H_k^TK_k^T-\Delta\underline{x}_k(-)\underline{v}_k^TK_k^T+\Delta\underline{x}_k(-)\pi_k^TK_k^T-K_kH_k\Delta\underline{x}_k(-)\Delta\underline{x}_k^T(-) \\
&-K_k\underline{v}_k\Delta\underline{x}_k^T(-)+K_k\pi_k\Delta\underline{x}_k^T(-)+K_k\{H_k\Delta\underline{x}_k(-)\Delta\underline{x}_k^T(-)H_k+H_k\Delta\underline{x}_k(-)\underline{v}_k^T-H_k\Delta\underline{x}_k(-)\pi_k^T+\underline{v}_k\Delta\underline{x}_k^T(-)H_k^T \\
&+\underline{v}_k\underline{v}_k^T-\underline{v}_k\pi_k^T-\underline{v}_k\pi_k^T-\pi_k\Delta\underline{x}_k^T(-)H_k^T-\pi_k\underline{v}_k^T+\pi_k\pi_k^T\}K_k^T]
\end{aligned} \tag{32}
$$

where, H_k denotes $TA_{BS}(q_c)H_K(\underline{\hat{x}}_k,\rho)$, π_k and H_k is a deterministic function, and if we neglect the third moment of $\Delta\underline{x}_k$ under the Gaussian assumption. Then Eq. (32) can be rewritten as follows:

$$
P_k(+)=P_k(-)-P_k(-)H_kK_k^T-K_kH_kP_k(-)+K_k\{H_kP_k(-)H_k^T+R_k\}K_k^T \tag{33}
$$

where, R_k is the covariance matrix with colored noise due to nonlinearity of measurement.

Let us define the cost function as in $Eq.$ (34) in order to obtain the filter gain to minimize the update covariance of $Eq.$ (33) (Bar-Itzhack and Oshman, 1985 ; Lewis, 1986).

$$J_k \equiv E[\Delta \underline{x}_k(+)\Delta \underline{x}_k^T(+)] \quad (34)$$

After substitution of *Eq*. (31) into *Eq*. (34) and differentiating *Eq*. (34) with respect to K_k, and putting it to zero, we can obtain the optimal gain under the Gaussian assumption. That is,

$$K_k = P_k(-)H_k^T\{H_kP_k(-)H_k^T+R_k\}^{-1} \quad (35)$$

Substituting *Eq*. (35) into *Eq*. (33), we can obtain the update covariance equation as follows:

$$P_k(+) = [I - K_kH_k]P_k(-) \quad (36)$$

We can find that *Eq*. (36) has the same form as the update covariance equation of the linear Kalman filter, and there is no error due to truncation errors because it is compensated by nonlinearity of the system and the measurement. However, because the derived update covariance equation neglects the third moment, the proposed nonlinear filter becomes a suboptimal filter that does not contain truncation errors due to nonlinearity.

5. Attitude Algorithm for Nonlinear Filter

The proposed nonlinear filter in section 4 requires a lot of computation because of an inherent nonlinearity and complexity of attitude dynamics with coupled terms. We introduce here a computation reduction technique using the method of state division for a real time implementation. We assume that the solution of the covariance *Eq*. (21) for propagation can be divided, according to the state $\underline{x}_1$ and state $\underline{x}_2$, by

$$P_k(-) = \begin{bmatrix} P_{x_1}(-) & P_{x_1x_2}(-) \\ P_{x_1x_2}^T(-) & P_{x_2}(-) \end{bmatrix} \quad (37)$$

Substituting *Eq*. (37) into *Eq*. (21), we can obtain the individual form of the covariance differential equation. That is,

$$\dot{P}_{x_1}(-) = 1/2\Omega(\underline{u}-\hat{\underline{x}}_2)P_{x_1}(-) + 1/2P_{x_1}(-)\Omega^T(\underline{u}-\hat{\underline{x}}_2) - 1/2\Gamma(\hat{\underline{x}}_1)P_{x_1x_2}^T(-) - 1/2P_{x_1x_2}(-)\Gamma^T(\hat{\underline{x}}_1) + 1/4\Gamma(\hat{\underline{x}}_1)Q_1\Gamma^T(\hat{\underline{x}}_1) + M \quad (38a)$$

$$\dot{P}_{x_1x_2}(-) = 1/2\Omega(\underline{u}-\hat{\underline{x}}_2)P_{x_1x_2}(-) - 1/2\Gamma(\hat{\underline{x}}_1)P_{x_2}(-) \quad (38b)$$

$$\dot{P}_{x_2}(-) = Q_2 \quad (38c)$$

Inspecting the three equations in *Eq*. (38), we can find that the solution of P_{x_2} can be obtained independently by *Eq*. (38c). Therefore, we can reduce computation time since the differential equations in *Eq*. (38) are solved using a sequential method as follows;

1) The solution of P_{x_2} can be obtained independently by *Eq*. (38c).

2) Substituting the solution of *Eq*. (38c) into *Eq*. (38b), the solution of $P_{x_1x_2}$ can be obtained independently by *Eq*. (38b).

3) Substituting the solution of *Eq*. (38b) into *Eq*. (38a), the solution of P_{x_1} can be obtained independently by *Eq*. (38a).

Also, we assume that K_k the filter gain in *Eq*. (35) is divided as in *Eq*. (38), and we subsititute *Eq*. (37) and *Eq*. (38) into *Eq*. (35). The divided filter gain equation is given by *Eq*. (39).

$$K_k = \begin{bmatrix} K_{x_1} \\ K_{x_2} \end{bmatrix} \quad (39)$$

$$K_k = \begin{bmatrix} P_{x_1}(-) & P_{x_1x_2}(-) \\ P_{x_1x_2}^T(-) & P_{x_2}(-) \end{bmatrix} H_k^T[H_kP_k(-)H_k^T+R]^{-1} \quad (40)$$

Inspecting *Eq*. (40), we can define H_k as in *Eq*. (41) since the measurement matrix H_k in *Eq*. (40) consists of only state $\underline{x}_1$.

$$H_k = [H_{x_1} \mid 0] \quad (41)$$

Substituting *Eq*. (41) into *Eq*. (36) and *Eq*. (40), the divided filter gain and update covariance are derived as follows:

$$K_{x_1} = P_{x_1}(-)H_{x_1}^T[H_{x_1}P_{x_1}(-)H_{x_1}^T + R]^{-1} \quad (42a)$$

$$K_{x_2} = P_{x_1x_2}^T(-)H_{x_1}^T[H_{x_1}P_{x_1}(-)H_{x_1}^T + R]^{-1} \quad (42b)$$

$$P_{x_1}(+) = P_{x_1}(-) - K_{x_1}H_{x_1}P_{x_1}(-) \quad (42c)$$

$$P_{x_1x_2}(+) = P_{x_1x_2}(-) - K_{x_1}H_{x_1}P_{x_1x_2}(-) \quad (42d)$$

$$P_{x_2}(+) = P_{x_2}(-) - K_{x_2}H_{x_1}P_{x_1x_2}(-) \quad (42e)$$

Inspecting the individual state update *Eq*. (42), we can independently solve them because they represent each of the elements of the divided state. Generally, computation of the matrix is proportional to its dimension cubed (Bar-Itzhack and Medan, 1983). The proposed attitude estimation algorithm is divided into two groups as in *Eq*. (38) and *Eq*. (42). If the dimension, n, of the system can be devided into two groups of dimension r and m which satisfy *Eq*. (38) and *Eq*.

(42), then the number of the reduced operator is $3(r^2m+rm^2)$. Since $n=6$, $r=3$, and $m=3$ in this paper for determining the attitude of the satellite, the proposed algorithm reduces the computation time approximate by 1/4 times. The proposed algorithm will be useful for a real time attitude estimation of a spacecraft since it requires less computing time compared with any existing nonlinear algorithms.

6. Simulation Results

Performance of the proposed attitude algorithm in section 5 is verified through a flow diagram as shown in Fig. 2. We assume that the orbital period of the spacecraft is 120 *min*., the spacecraft has two star trackers, and the angular velocity of spacecraft sensed by the gyros is 0.05*deq/sec*. Hence, $\omega_1=0$, $\omega_2=0.005 deg/sec$, and $\omega_3=0 deg/sec$ are used for the simulation. Recently the attitude sensors, when operating under high and continuous slew rates, acceleration, and jerk motions, may introduce significant errors because of coupled terms in the three axes of the spacecraft. Therefore, to verify the performance of the proposed algorithm and the standard EKF, simulations are performed by varying the initial state values of the filters as in Table 1. Also, the initial covariance values corresponding to the initial state values are given as follows:

$$\delta\phi_0=5^\circ\ ;\ P_0=\begin{bmatrix} 6.4\times10^{-4}I_{4\times4} & 0 \\ 0 & 5.3\times10^{-11}I_{3\times3}\end{bmatrix} \tag{43a}$$

Table 1 Initial values of the filters.

Rotation angle error	Quaternion error ($\delta\underline{q}$)
5deg	$\delta\underline{q}=(0.025, 0.025, 0.025)^T$
10deg	$\delta\underline{q}=(0.05, 0.05, 0.05)^T$
15deg	$\delta\underline{q}=(0.075, 0.075, 0.075)^T$

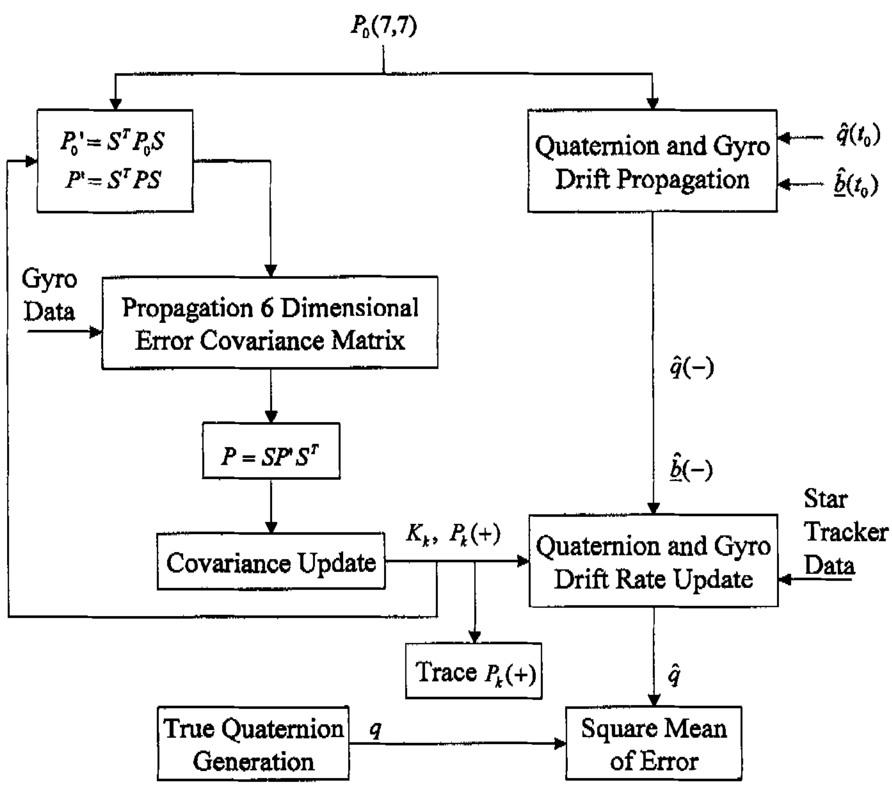

Fig. 2 Simulation flow diagram for the attitude estimation.

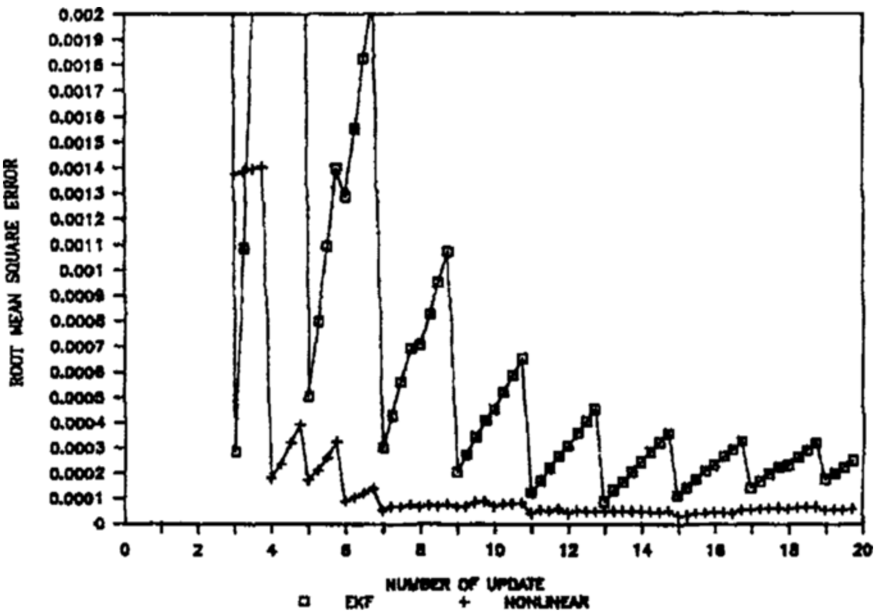

Fig. 3 The errors of the quaternion for the EKF and the nonlinear filter($\delta\phi_0$; 5°, R' ; 100).

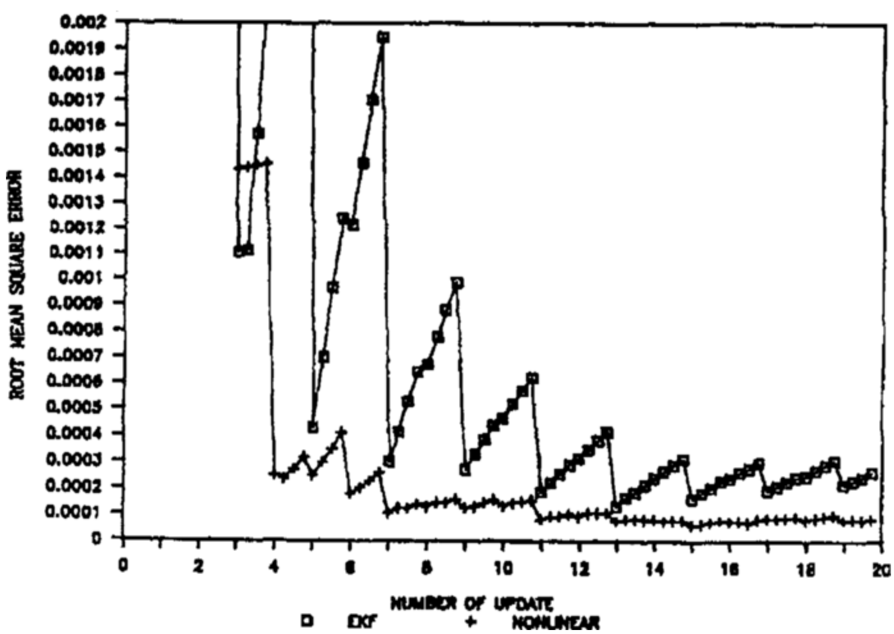

Fig. 4 The errors of the quaternion for the EKF and the nonlinear filter($\delta\phi_0$; 5°, R' ; 400).

$$\delta\phi_0 = 10° \ ; \ P_0 = \begin{bmatrix} 2.6\times10^{-3} I_{4\times4} & 0 \\ 0 & 5.3\times10^{-11} I_{3\times3} \end{bmatrix} \tag{43b}$$

$$\delta\phi_0 = 15° \ ; \ P_0 = \begin{bmatrix} 5.7\times10^{-3} I_{4\times4} & 0 \\ 0 & 5.3\times10^{-11} I_{3\times3} \end{bmatrix} \tag{43c}$$

The gyro noise and the measurement noise have been simulated using RAND and GAUSS subroutines that generate uniformly distributed random numbers and Gaussian-distributed random numbers, respectively. The gyro data was simulated for a sampling time of 500m *sec* and star

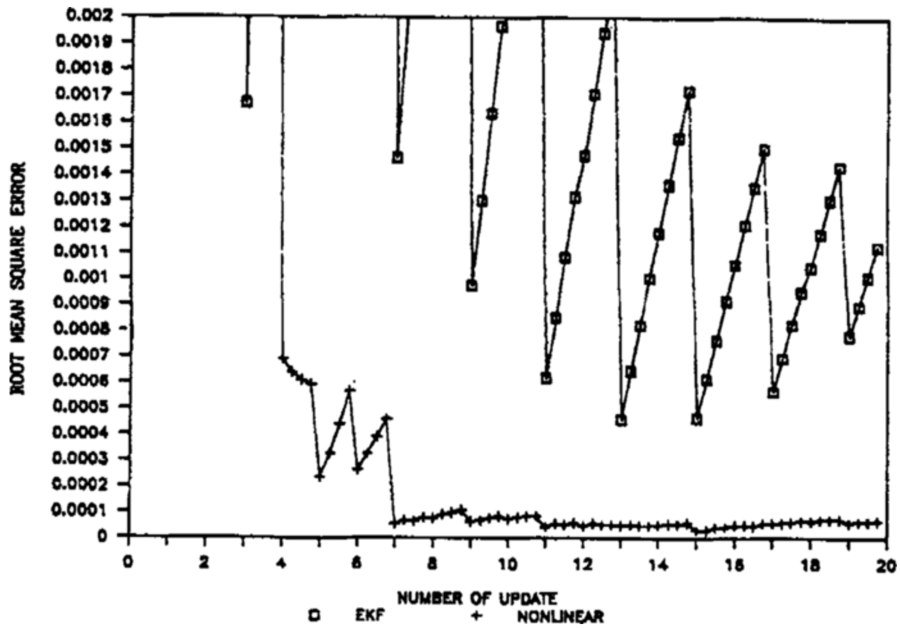

Fig. 5 The errors of the quaternion for the EKF and the nonlinear filter($\delta\phi_0$; 10°, R' ; 100).

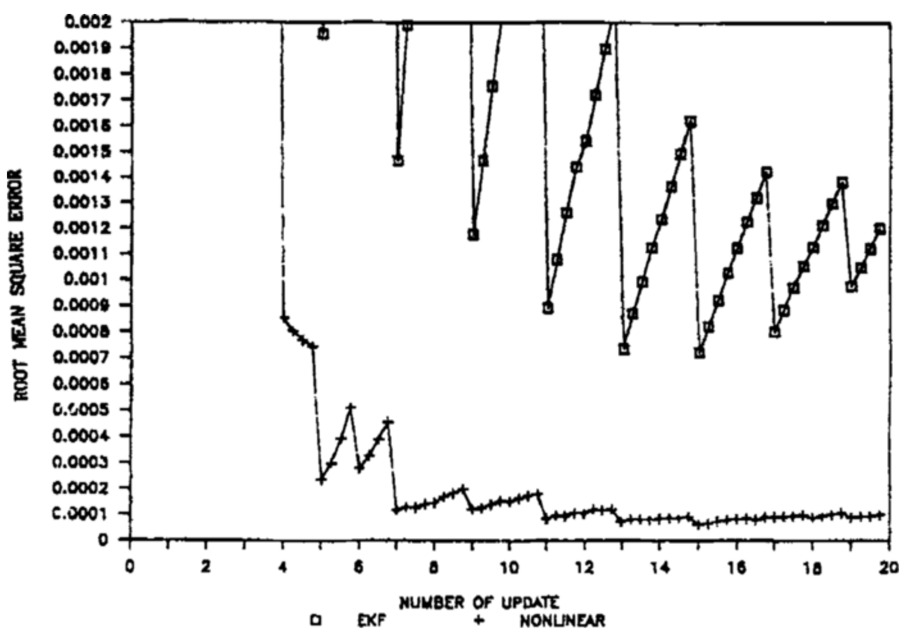

Fig. 6 The errors of the quaternion for the EKF and the nonlinear filter($\delta\phi_0$; 10°, R' ; 400).

tracker outputs were simulated at an interval of 120 *sec*. The covariance propagation equations have been simulated with a step size of 500m *sec* using a fourth-order Runge-Kutta scheme of numerical integration on the digital computer. The standard deviation of the process noise η_1 was simulated for 1 *arc second/sec*. The standard deviation of measurement noise R' was assumed to lie between 10 and 200 *arc seconds*. The standard deviation of the drift rate noise η_2 was assumed to be 4.7×10^{-5}*arc seconds/sec*.

Both the EKF and the proposed algorithm

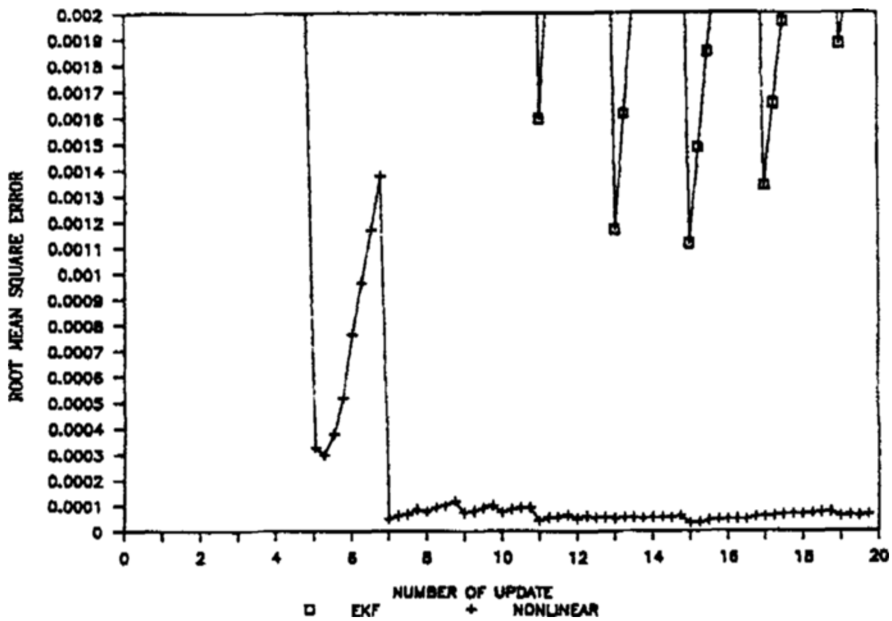

Fig. 7 The errors of the quaternion for the EKF and the nonlinear filter($\delta\phi_0$; 15°, R' ; 100).

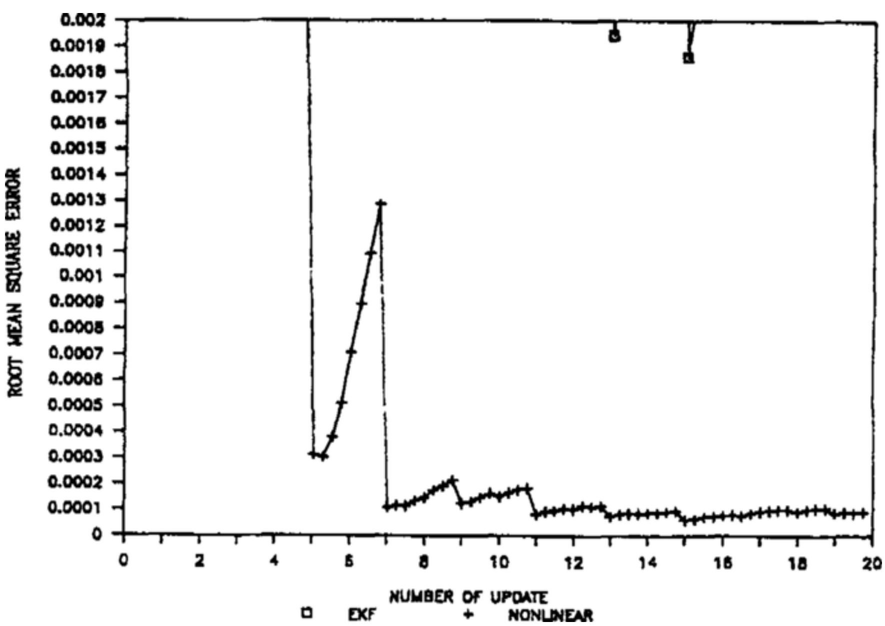

Fig. 8 The errors of the quaternion for the EKF and the nonlinear filter($\delta\phi_0$; 15°, R' ; 400).

were simulated under the given conditions. Since the quaternion estimations are random processes, 100 Monte Carlo simulation runs were carried out for the estimation algorithms. Many simulation runs have been made and the results are summarized in Fig. 3 to Fig. 8. The root-mean-square estimation errors of the quaternion against the star update are plotted in the figures. It can be seen from the figures that the proposed algorithm shows consistently better performance than that of the EKF in all the ranges of the initial state values and the covariance values of measurement.

From Fig. 3 and Fig. 4, it is apparent that the root-mean-square estimation errors of the quaternion are bounded by the measurement update, however, the root-mean-square estimation errors of the proposed algorithm are much lower than that of the EKF, and the convergence speed of the proposed algorithm is faster than the EKF. The initial value of the rotation vector error in Fig. 5 and Fig. 6 is 10 *deg*, and the initial value of the rotation vector error in Fig. 7 and Fig. 8 is 15 *deg*. It can be seen from Fig 5 to Fig. 8 that the performance of the EKF is not improved by the measurement update because of the model errors in the EKF. But the proposed algorithm exhibits an improved performance since the covariance of the filter is compensated by nonlinearities in the system.

7. Conclusions

The attitude algorithm presented in this paper deals with the problem of high and continuous maneuver base motion in applications where an accurate attitude estimation is required. The nonlinear filter for attitude estimation derived in this paper is accomplished by implicating the mean and covariance of nonlinearities in system and measurement. The derived nonlinear filter is a suboptimal estimator. However, the proposed estimator requires a lot of computation because of an inherent nonlinearity and complexity of the system model for attitude. For more efficient computation, this paper introduces a new attitude estimation algorithm using the state divided technique for a real time processing which is developed to provide accurate attitude determination capability under the highly maneuvering dynamic environment. The proposed estimation algorithm for spacecraft attitude does not produce any truncation bias errors, and it does not diverge because the covariance of the estimator is compensated by the nonlinear terms of the system. Therefore the proposed estimator exhibits an improved performance compared with the EKF.

To verify the performance of the proposed algorithm with reference to the EKF, simulations were carried out for several initial values of the state and covariance and several measurement covariance. Simulation results show that the proposed algorithm had consistently better performance than that of the EKF for all of the cases.

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